

Spacelike Singularities and String Theory

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Abstract

An interpretation of spacelike singularities in string theory uses target space duality to relate the collapsing Schwarzschild geometry near the singularity to an inflationary cosmology in dual variables. An appealing picture thus results whereby gravitational collapse seeds the formation of a new universe.

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String theory is strongly believed to solve the short-distance problems of quantum gravity by providing a fundamental length scale $\ell_{str} = \sqrt{\hbar c/T}$, where T is the string tension. Perturbative studies of high-energy string scattering (Gross and Mende 1988, Amati *et.al.* 1988), the stringy resolution of orbifold singularities (Dixon *et.al.* 1985), and target-space duality (for a review, see Giveon *et.al.* 1994) all point in this direction. Thus one might expect that there is no such thing as a ‘naked’ or timelike singularity in string theory; the exponentially soft high energy behavior of the theory (in weak coupling, at least) will forbid any observation of the kinds of arbitrarily violent processes associated with high field gradients over small regions. The decay of strong electromagnetic fields by string pair production (Bachas and Porrati 1992; see also Fradkin and Tseytlin 1985, Russo and Tseytlin 1994) also supports this picture of the string’s aversion to large field strengths. Spacelike singularities, such as those encountered in black hole formation, seem to mount a stronger challenge. In this case, the singularity is in the future; all the soft high-energy behavior is helpless in the face of the crushing together of future-directed light cones inside the black hole horizon. No causal force can prevent collapse; indeed the soft high-energy behavior of strings at high energy would seem to act in the wrong direction, placing an upper limit on any potential stabilizing force. How then can string theory be a complete theory of nature if it does not tell us

how to evolve geometry through a spacelike singularity? I will argue here that this question presupposes an improper treatment of string geometry near the singularity.

Target-space duality (often called T-duality) appears to be a pervasive feature of classical solutions in string theory (*c.f.* Giveon *et.al.* 1994); it has been demonstrated in toroidal and orbifold geometries (Kikkawa and Yamasaki 1984, Sakai and Senda 1986, Nair *et.al.* 1987), cosmological settings (*c.f.* Tseytlin and Vafa 1992, Veneziano 1991), and even the moduli space of $K3$ surfaces (Aspinwall and Morrison 1994; see also Seiberg 1988). The use of mirror symmetry in the last case indicates a similar duality of Calabi-Yau threefolds. In all these instances, no regime exists in which the proper low-energy interpretation of the geometry involves a manifold with a scale size $R \ll \ell_{str}$. Rather, as the geometry is taken below scale sizes of order ℓ_{str} , the low-energy effective lagrangian typically breaks down via the appearance of new soft modes (*e.g.* winding modes in toroidal and orbifold geometries); simultaneously, the modes of the original low-energy fields become stiff. Decreasing the scale size further, the original low-energy fields have momenta larger than $1/\ell_{str}$ and should no longer be included in the effective action, whereas the new soft modes become even softer and comprise the fields with which low-energy dynamics is properly understood. This new dynamics can be dramatically different from that of the original effective field theory; it can change the low-energy field content (Dine *et.al.* 1989), gauge group (Narain *et.al.* 1987, Ginsparg 1987), and even the spacetime topology (Kiritsis and Kounnas 1994).

With this characteristic of string geometry in mind, let us examine the Schwarzschild solution (the analysis applies without essential modification to cosmological collapse situations). Suppose that string theory is weakly coupled: $M_{pl} = M_{str}/g_{str}$ with the string coupling $e^{-\Phi} = g_{str} \ll 1$; and consider large black holes $M \gg M_{str}$ (the Schwarzschild radius $R = 2g^2\ell_{str}^2 M \gg \ell_{str}$). The line element is

$$ds^2 = -[1 - (\frac{2M}{r})^{d-3}]dt^2 + [1 - (\frac{2M}{r})^{d-3}]^{-1}dr^2 + r^2 d\Omega_{d-2}^2. \quad (1)$$

Inside the horizon, t is a spacelike coordinate whereas r is timelike. Therefore let us define $\tau = [2R/(d-1)](r/R)^{(d-1)/2}$ and $\rho = t$, so that near the singularity

$$\begin{aligned} ds^2 &\sim -d\tau^2 + a(\tau) d\rho^2 + b(\tau) d\Omega_{d-2}^2 \\ &= -d\tau^2 + \left[\frac{(d-1)\tau}{2R}\right]^{-2(d-3)/(d-1)} d\rho^2 + R^2 \left[\frac{(d-1)\tau}{2R}\right]^{4/(d-1)} d\Omega_{d-2}^2. \end{aligned} \quad (2)$$

One can regard this metric as an anisotropic homogeneous cosmological model with spatial sections of topology $\mathbb{R} \times \mathbf{S}^{d-2}$ (*c.f.* MacCallum 1979 for a review). As the singularity approaches, the scale factor of the \mathbb{R} direction stretches to infinity while the radius of the \mathbf{S}^{d-2} shrinks to zero. However, at $\tau \sim \ell_{str}$ – a thickened hypersurface we shall call the ‘threshold’ for lack of a better term¹

¹Perhaps ‘stretched singularity’ would conform more to current usage; purgatory (‘a place

– we should switch the description to a dual effective action of the \mathbf{S}^{d-2} . Very crudely speaking, we should take

$$b(\tau) \rightarrow 1/b(\tau) \quad (3)$$

while shifting the string coupling

$$\Phi \rightarrow \Phi - (d-2) \log b . \quad (4)$$

In this regime the kinetic terms (the extrinsic curvature of spatial sections) dominate over potential terms (intrinsic spatial curvature) in the Hamiltonian constraint equation; hence to a first approximation we can perhaps ignore the curvature of the sphere and borrow the familiar duality transformation properties (3),(4) of flat (toroidal) geometries. Of course, large field gradients will strongly modify the specific form of the duality transformation. Our strategy will be to build confidence in situations where we can make reliable perturbative expansions, and then extrapolate to the regime of rapidly varying scale factors.

For spatial topology $\mathbb{R} \times (\mathbf{S}^1)^{(d-2)}$ carrying metrics of the form

$$ds^2 = -dt^2 + \sum_i (a_i(t) dx_i)^2 , \quad (5)$$

equations (3),(4) are indeed the correct duality transformations for slowly evolving radii a_i (Veneziano 1991). If a_1 (the \mathbb{R} scale factor) is large and increasing while a_2, \dots, a_{d-1} decrease from $a > \ell_{str}$ to $a < \ell_{str}$, we would have no qualms about changing the low energy fields from momentum to winding modes at $a \sim \ell_{str}$. We can make this evolution adiabatic by allowing the dilaton field Φ to evolve in time (see *e.g.* Tseytlin 1992c, Kiritsis and Kounnas 1994). Since the target space duality symmetry in the toroidal case is due to an easily identifiable structure (symmetry between momentum and winding number), we can plausibly expect duality to persist even during rapid changes of the radii – for instance near the Kasner-type singularity (*c.f.* MacCallum 1979)

$$ds^2 = -\ell_{str}^2 d\tau^2 + \tau^{-2(d-3)/(d-1)} dx_1^2 + \tau^{4/(d-1)} \sum_{i=2}^{d-1} dx_i^2 \quad (6)$$

analogous to (2). Thus for $\tau \lesssim 1$ we should switch to the dual theory (3),(4):

$$\begin{aligned} \tilde{ds}^2 &\sim -\ell_{str}^2 d\tau^2 + \tau^{-2(d-3)/(d-1)} dx_1^2 + \tau^{-4/(d-1)} \sum_{i=2}^{d-1} dx_i^2 \\ \Phi &\sim \log g_{str,0} - 2 \left(\frac{d-2}{d-1} \right) \log \tau . \end{aligned} \quad (7)$$

or state of temporary suffering or misery' – Webster's ninth New Collegiate Dictionary) might also be an apt description of this region.

This metric describes a so-called super-inflationary universe (inflation to infinite scale factor in finite comoving time (Lucchin and Matarrese 1985)). The inflation is driven by the kinetic energy of the dilaton field, which is running toward strong coupling.

Exact vacuum solutions of string theory may be found in the similar situation of gravitational plane wave solutions (Horowitz and Steif 1990, Tseytlin 1992b, Polchinski and Smith 1991), where a_i evolve as a function of a null coordinate u . One can choose the wave profile $a_i(u)$ to evolve between any given initial and final values as the wave passes; in particular there is no restriction on the gradient of the scale factor. The string coupling can be made to change by a finite amount across the wave, hence the theory can be made arbitrarily weakly coupled. One can even contemplate a null singularity, where the metric blows up at finite u (Horowitz and Steif 1990, de Vega and Sanchez 1992) – a kind of null version of the Schwarzschild singularity. An analysis of this situation is in progress.

Generically, a dilaton rolling toward weak coupling provides a friction that damps the growth of the scale factor, while evolution to strong coupling has a runaway behaviour. The minisuperspace cosmological equations read (*c.f.* Tseytlin and Vafa 1992, Tseytlin 1992a, Veneziano 1991)

$$\begin{aligned}\dot{\varphi}^2 - \sum_{i=1}^{d-1} \dot{\lambda}_i^2 &= 2U + 2e^\varphi E \\ \ddot{\lambda}_i - \dot{\varphi} \dot{\lambda}_i &= -\frac{\partial U}{\partial \lambda_i} + e^\varphi P_i \\ \ddot{\varphi} - \sum_{i=1}^{d-1} \dot{\lambda}_i^2 &= \frac{\partial U}{\partial \varphi} + e^\varphi E\end{aligned}\tag{8}$$

in terms of the shifted dilaton $\varphi = 2\Phi - \sum_i \lambda_i$ and the scale factor $\lambda_i = \log(a_i)$; $U(\varphi, \lambda_i)$ is the effective potential and E, P_i are the energy and pressure of a perfect fluid of string matter. One easily sees from these equations that $\dot{\varphi} < 0$ damps the expansion of spatial volume, which then freezes the dilaton. On the other hand, $\dot{\varphi} > 0$ is unstable to growing spatial volume, which then accelerates the growth of the dilaton, *etc.*, generating super-inflation. Depending on the form of matter stress-energy and its equation of state, one can obtain a number of different behaviors including oscillatory scale factors (Tseytlin 1992c). An appealing scenario has an initial era of dilaton-driven inflation; then the dilaton freezes or settles toward a weak-coupling regime while string matter generated during inflation acts as a source for further cosmological expansion. Strings in a (super)inflationary universe are unstable in the sense that the string size grows as the scale factor (Gasperini *et.al.* 1991). If the inflationary era terminates, the produced strings might serve as a source of further cosmological expansion (*c.f.* Turok 1988, Barrow 1988) as well as the seeds for structure formation. I must emphasize that the T-dual effective lagrangian will not be quantitatively valid

in the regime we wish to use it, since the metric and dilaton are changing rapidly on scales of order ℓ_{str} . Nevertheless, we may hope to borrow intuition gained from model situations (like the plane wave solutions) where one can continuously vary the geometry between gentle and violent evolution of the scale factors.

Unfortunately, at present one can only speculate on the mechanism, if any, that terminates this dilaton-driven inflation (for a discussion of the problems, see Brustein and Veneziano 1994). Perhaps, at sufficient dilaton velocity, higher string corrections to the equations of motion absorb the dilaton kinetic energy driving the expansion, and turn the dilaton evolution back toward weak coupling. Alternatively, if strong-weak coupling duality (so-called S-duality) is a feature of string theory, once the inner, T-dual universe inflates to the strong coupling regime, we change effective descriptions again so that the theory is headed toward weak coupling in the ST-dual theory with interaction strength $1/g_{str}$ (*c.f.* Horne and Moore 1994). It must be emphasized that evidence for strong-weak coupling duality in string theory is meager at best. Any quantity receiving perturbative corrections cannot be tested without an understanding of nonperturbative string dynamics (*c.f.* Sen 1994). Hence one can as yet investigate only renormalizable gauge theories having enough supersymmetries that classical relations are perturbatively exact for the quantity of interest (Sen 1994, Seiberg and Witten 1994; for intriguing hints of what may lie beyond, see Gauntlett and Harvey 1994). One thus cannot be sure whether S-duality is a property of string theory or merely of this restricted class of gauge theories. In this context, I should point out that the form of the low-energy effective action which manifests S-duality is written using the Einstein metric G_{ab}^E for which it is natural to think of $M_{pl} = M_{str}e^{-\Phi}$ as fixed. However for string theory it is more natural to use the sigma-model metric $G_{ab}^\sigma = e^{\alpha\Phi}G_{ab}^E$ in which the fundamental constant M_{str} is fixed. Hence the S-dual of a string theory with $M_{pl} = M_{str}e^{-\Phi}$ will be one with $M_{pl} = M_{str}e^{+\Phi}$. Curiously the latter is precisely the relation between the string and Planck scales in open string theory if the former is that of heterotic strings (Dine and Seiberg 1985; see also Polchinski 1994), suggesting that S-duality might relate rather different sorts of weakly coupled string theories if it holds.

We expect phenomena quite similar to the toroidal example to occur in the $\mathbb{R} \times \mathbf{S}^{d-2}$ topology relevant to gravitational collapse. The main difference lies in the fact that T-duality is a more complicated transformation in this case, mixing all the string modes even for an adiabatically changing scale factor. There is no clean separation between momentum and ‘winding’ modes. Nevertheless, we expect soft modes arising from the fact that entropy of string wandering is less suppressed than center of mass momentum at scale factors much smaller than ℓ_{str} . By maintaining a wide separation of scales $\ell_{str} \gg \ell_{pl}$, we expect that the transition to the dual geometry occurs before quantum gravity effects become important. This is not to say that quantum effects will be negligible before the coupling grows strong. Curvatures and field strengths are of order the string scale in the transition region and in the dual geometry, so string

creation will have an important effect on the subsequent expansion in the T-dual theory. For instance, Zel'dovich (1970) has argued that particle creation rapidly isotropizes the expansion of initially anisotropic cosmological models such as (7). One can imagine that the aversion to high field strength of the sort exhibited in the situation of constant electromagnetic fields (Bachas and Porrati 1992, Russo and Tseytlin 1994) could provide a drag on the expansion through copious string production. The production in a classical background is $O(\hbar)$ but independent of g_{str} at lowest order, so it can be large when field strengths are $O(1)$ in natural string units even when g_{str} is small. In the electromagnetic field example, magnetic fields in excess of a critical value cause a number of string modes to become light and then tachyonic; critical electric fields cause the pair production rate for an infinite number of modes to diverge. The back reaction of the produced pairs rapidly acts to dissipate the field. In the gravitational case string production should provide energy density to slow the expansion to a rate of order one in string units. When the expansion rate exceeds one in natural string units, tidal forces exceed the string tension and any string grows as the scale factor. There is no obvious quantum number that prevents spontaneous tree-level generation of strings – for instance zero momentum dilatons – which then start to grow without bound. The produced strings' energy density will act to slow expansion.

In fact, curvature becomes large already in the Schwarzschild region at a $(d-2)$ -sphere radius of order $r_0 \sim R(\ell_{str}/R)^{2/(d-1)} \gg \ell_{str}$; thus one might expect strong quantum effects *before* the $(d-2)$ -spheres become small. Several groups (Israel and Poisson 1988, Frolov *et.al.* (1990)) have speculated that strong quantum particle production causes a transition to an inflationary de Sitter phase, thereby avoiding gravitational collapse in a manner quite similar to the present proposal but without resorting to string theory (although since their scenario is particle-theoretic, the subsequent 'inner' universe is made out of ordinary matter rather than stringy dual matter; also ℓ_{str} in our considerations is replaced by ℓ_{pl} in theirs). However, the elapsed proper time between spheres of radius r_0 and radius ℓ_{str} is the string scale ℓ_{str}/c . To realize this de Sitter transition, quantum effects would need to *reverse* the momentum of the geometry (the extrinsic curvature of spatial hypersurfaces) over an essentially meaningless time scale of order one in string units; whereas all that need be done to control the curvature is to *slow the rate of change* of the momentum. Whether quantum effects avert, enhance or merely delay gravitational collapse remains to be seen, and will require a better understanding of string geometry at large field gradients. String theory offers the possibility that the collapse need not be stopped, with field momentum built up during collapse used to fuel expansion into a large universe using new degrees of freedom unavailable within the realm of field theory.

Let us now extend the scope of our speculations to the issue of black hole evaporation (Hawking 1975). We only expect the metric (2) to apply outside the collapsing matter and until the endpoint of the Hawking evaporation process,

i.e. in an interval $\rho(\text{formation}) < \rho < \rho(\text{endpoint})$. At $\rho(\text{endpt})$, an accelerating accumulation of negative stress-energy – heuristically the partners of the radiated Hawking particles – strikes the ‘threshold’, which might then turn timelike. The spacelike transition region to the dual cosmology is bracketed by two implosive events; the nearly null collapse of infalling matter, and the final flash of radiation at the endpoint. What could the geometry look like afterward? The original spacetime will be Schwarzschild (together with outgoing Hawking radiation) down to a radius of order ℓ_{str} . The cosmology of the dual universe beyond the threshold will presumably continue to evolve. There will then presumably be a spacelike path of stringy dimensions through a wormhole connecting the two regions. It is unlikely that the threshold could pinch off and disconnect the two worlds; this would require precisely the kind of singularity that we have argued should not occur in string theory. The geometry has features of a stable remnant from the viewpoint of the original (Schwarzschild) universe, as well as of baby universe formation; both objects have been considered in the context of the evaporation process (Aharonov *et.al.* 1987, Banks *et.al.* 1992; Zel’dovich 1977, Dyson 1976). For a crude picture of the geometry, see figure 1 (it is assumed that inflation of the dual geometry terminates at some point). Because of the large volume of the dual geometry beyond the (now timelike) threshold, this sort of remnant should be hard to pair produce (Banks *et.al.* 1993).

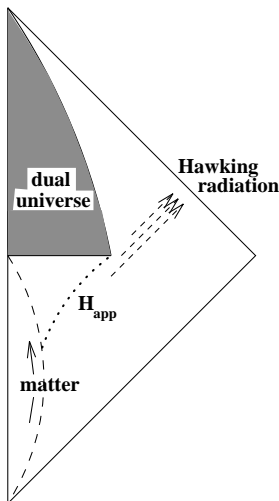


Figure 1
Penrose diagram for a black hole which forms and subsequently evaporates. The dashed line indicates the trajectory of collapsing matter; the dotted line the apparent horizon.

One can again imagine adiabatic transition regions that could serve as a

model for how the geometry might appear well after the endpoint. Consider again the toroidal universe (7), but now allow the torus radii $\lambda_i = \log(a_i/\ell_{str})$ to vary adiabatically in the noncompact x_1 direction about a central value of zero (*i.e.* $a_i \sim \ell_{str}$). For instance we can consider a ‘standing wave’ $\lambda_2 = \log(R_{\max}) \cos(x_1/L) \cos(t/L)$ (see figure 2). This will not satisfy the string equations of motion, but we may compensate by allowing adiabatic variation of some of the other fields (similar solutions are described in Kiritsis and Kounnas (1994)). An observer at $x_1 = 0$ will see a large $(d-1)$ -dimensional spatial universe; generic objects in his world carry momentum in the x_2 direction, which is nearly continuous. However the gap in p_2 gets large near $x_1 = \pi/2$ and the spatial world looks effectively $(d-2)$ -dimensional with a spatially varying mass $M_{d-2} = p_2 \exp[-\lambda_2(x_1, t)/\ell_{str}]$. From the $(d-1)$ -dimensional viewpoint the geometry is conical in this region; there is an ‘angular momentum barrier’ to probing the tip of the cone. A test particle carrying nonvanishing p_2 sent to probe the world at $x_1 = L\pi/2$ will be reflected by this effective potential. It cannot penetrate into the dual world because it is a macroscopic winding string there with energy of order R_{\max}/ℓ_{str}^2 . The observer’s world is effectively of finite extent $L\pi$ in the x_1 direction. He sees a ‘big bang’ in his past at $t = -L\pi/2$ and will experience a ‘big crunch’ at $t = L\pi/2$ when he will be stretched on the cosmic rack of the dual world that follows. Observers in this universe cannot look past their cosmological ‘singularity’ because their effective lagrangian cannot describe it. Only the nongeneric modes with $p_2 = 0$ can pass into the dual world, like the S-waves of GUT monopoles (Callan 1982; Rubakov 1981). One might expect a ‘threshold’ remnant to have similar properties: only very special modes can pass into or out of the dual world, and for most purposes it would look to the Schwarzschild observer as a soliton of mass $O(M_{str})$.

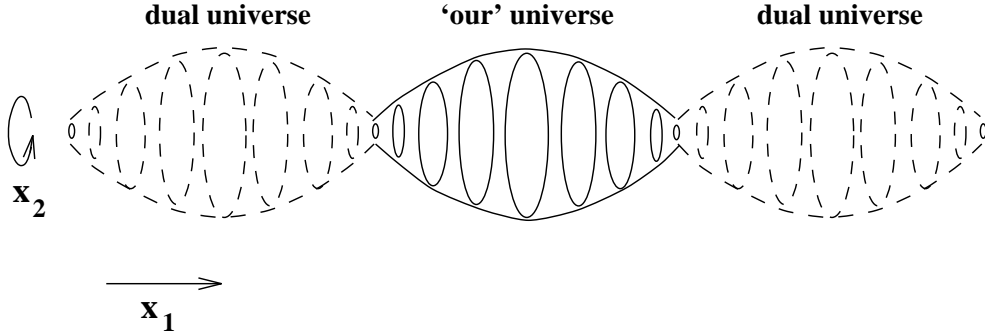


Figure 2
Toroidal model of a remnant; a transition region forms a ‘domain wall’ between two regions of spacetime described by T-dual geometries. Reading from left to right gives a picture of the evolution in t for fixed x_1 .

Unfortunately, there is no obvious barrier to bidirectional energy flow across a timelike threshold. What prevents the hot dual string gas beyond the threshold from leaking into our universe, violating energy conservation? The answer may require good control over the geometry in the threshold region to see if a barrier might arise; perhaps the neutral tubular ‘remnant’ described by Giddings *et.al.* (1993) could provide a useful starting point. Alternatively, the problem can be avoided altogether if the threshold always lies to the future of the exterior Schwarzschild region. This occurs when the threshold asymptotes to a null curve (figure 3) as the evaporation comes to an end.²

²A possibility suggested to me by M. O’Loughlin.

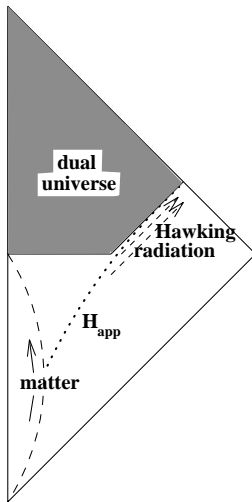


Figure 3
Alternative scenario for black hole evaporation, where the threshold asymptotes to a null trajectory.

We have explored a T-duality model of collapse and subsequent re-expansion, but there may well be other models. In the algebraic geometry of Calabi-Yau sigma models, there are paths in the Kähler moduli space (Witten 1993, Aspinwall *et.al.* 1994) along which a homological two-sphere shrinks to ‘zero’ size, then expands to positive radius in another geometry. From the viewpoint of the original low-energy description, the two sphere evolves to *negative* radius. The dilaton expectation value remains constant along the path. In gravitational collapse, it may be that the ‘zero radius’ boundary in the space of metrics on the collapsing \mathbf{S}^{d-2} joins smoothly onto another geometry in which it is expanding. The key feature to both this model and the T-duality model of collapse is the re-expansion toward large spatial volume in a different semi-classical geometry.

The property of string theory that we want to exploit is the absence of geometries with scale size smaller than ℓ_{str} . Singularity of the geometry is avoided simply if the collapsing spheres never reach a singular state; although perhaps somewhat more aesthetically pleasing from a cosmological standpoint, re-expansion is not essential. The \mathbf{S}^{d-2} could stabilize at some small radius of order ℓ_{str} as in the ‘cornucopion’ models of Banks *et.al.* (1992). If we regard τ in (2) as renormalization group time, the results of Cecotti and Vafa (1992) point toward the stabilization of the geometry of N=2 supersymmetric sigma models as they continue through ‘zero radius’; however in the \mathbf{CP}^n models they studied in detail, it is not apparent whether the geometry reexpands. If it does

not, one would then need to explain how the kinetic energy of the collapsing geometry is dissipated.

To summarize, target space duality (or some similar model of string geometry) may offer a means to discover the manner in which string theory resolves spacelike singularities. At the very least, it indicates that all issues of singularities in string theory might be recast as questions of (perhaps strong coupling) dynamics in large spatial volumes rather than problems of infinite field strength and consequent breakdown of dynamics.

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Note added

After this work was completed, E. Kiritsis and C. Kounnas informed me that they had also proposed target space duality as a means of resolving black hole singularities (*c.f.* Kiritsis 1993). I would like to thank them for bringing this work to my attention.

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